Observation of thermal fluctuations of disclination lines in a nematic liquid crystal

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We present measurements of the line tension and the viscous drag of a disclination line in nematic liquid crystal via observation of thermal fluctuations of the line position. Fourier analysis of the displacement of a disclination line due to thermal fluctuations was used to obtain the amplitudes and the relaxation rates of eigenmodes of the fluctuations. From them the line tension and the viscous drag were deduced. Our measurements show the coupling between flow and nematic orientation affects the viscous drag considerably, while the values of the line tension agree with a simple theory already predicted by de Gennes.

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Topological defects play an important role in ordered condensed matter and affect its properties (e.g., vortices in type II superconductors and in superfluids, defects in liquid crystals . . .). In liquid crystals they occur during symmetry breaking phase transitions, under external fields or they are due to impurities or confinement of liquid crystal in films, pores, etc. with frustrating surface anchoring [1,2]. The simplest of liquid crystalline phases—nematic phase got its name from threadlike structures—disclinations. These are topological line defects that can move, annihilate, decay into more defect lines, they accompany colloidal particles [3], get pinned to impurities or irregularities of the surface. Their static properties have been thoroughly studied and are well understood, while recent studies focus more on their hydrodynamical properties [4–6].

Hydrodynamical behavior of disclinations is governed by the viscoelastic properties of the medium, which can be reduced to two simple properties of the disclination itself, i.e., the line tension, which essentially tells the energy per unit length, and the viscous drag, i.e., the viscous drag force per unit length. They determine the dynamical behavior of the line (e.g., motion under external influence, annihilation of lines, relaxation to equilibrium, . . .). While there are different theoretical predictions for the line tension [1,7] and the viscous drag [8,9], only rare measurements of the line tension exist [10], and to our knowledge none for the viscous drag. In this paper we present measurements of both, the line tension and the viscous drag and their temperature dependence, via observation of thermal fluctuations of the line position.

Thermal fluctuations are a ubiquitous phenomenon which is particularly pronounced in liquid crystals. Well known fluctuations of the nematic order parameter are responsible for turbid appearance of nematic liquid crystals. They affect critical phenomena at phase transitions [1] and Freedericksz transitions [11]. Recently it has also been shown that the decay of the defects with higher order is accompanied by critical fluctuations [12]. Disclination lines are objects that can be considered as massless strings with given line tension and viscous drag and are also subject to thermal fluctuations. In this contribution we present a study of fluctuating modes of disclination lines using polarization microscopy. This is a unique way to determine both the line tension and the viscous drag of a disclination line.

Nematic liquid crystal 4-pentyl-4′-cyanobiphenyl (5CB) was sandwiched between two glass plates with no extra surface treatment. The anchoring was planar with no specific preferred direction. The cell thickness was 120 μm. Under polarizing microscope we were able to observe large areas of planarly oriented ordered nematic liquid crystals and many defect lines. Many of the line defects were pinned to one of the glass plates, but some of the lines went from one plate to another with the ends pinned to the glass plates. These were disclinations of strength 1/2 or $-1/2$. Polarization microscope with a charge-coupled device camera was used to study the motion of such disclination lines. Inspection of time sequences showed that the disclination lines exhibit random motion, i.e., thermal fluctuations.

One can think of a disclination line as a massless string for which the dynamic equation is

$$\sigma \frac{\partial^2 s_j}{\partial x^2} - R \frac{\partial s_j}{\partial t} = 0, \tag{1}$$

where s_j $(j=y,z)$ is the displacement of the line, σ is the line tension, and R is the viscous drag. These two coefficients depend on the detailed structure of nematic order parameter around the line and in its core. The eigenmodes of such a string are overdamped sinusoidal modes with wavevectors $q_n = n\pi/l$, where l is the length of the disclination line, provided that the line is firmly fixed at the ends. Relaxation rates of the modes depend on both the line tension and the viscous drag

$$\frac{1}{\tau_n} = \frac{\sigma}{R} q_n^2. \tag{2}$$

For each wave vector there are two modes with different polarizations corresponding to s_j $(j=y,z)$. In one elastic constant approximation these two modes are degenerate, so in this approximation it is enough to study only one polarization as we do in our experiment. In the rest of the paper the index j in s_j is omitted.

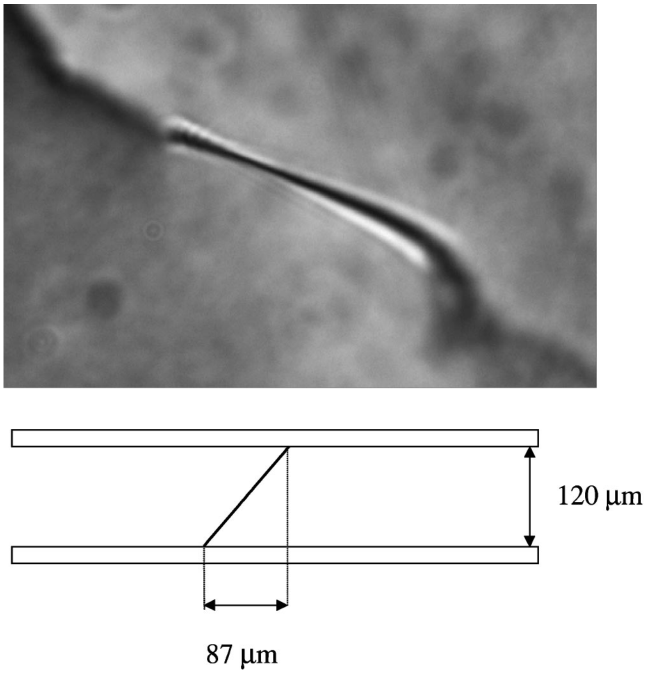

FIG. 1. Polarizing miscroscope image of a disclination with a length of 147 μm. (Numerical aperture is 0.4.) Schematic side view is shown below.

In Fig. 1 an image of a disclination line that goes from one glass plate to another with a schematic side view is shown. Sequences of 100–1000 images were taken with time delays between successive images of 100 ms or 200 ms and exposure time of 32 ms. Single disclination lines were then analyzed. By fitting the cross section of the line in the image with a Gaussian function the position of the line $\mathbf{r}_i(t)$ at different times t was determined, where i denotes the ith point of the line (the points are approximately one pixel, i.e., depending on resolution, 0.33 or 0.165 μ m apart). Then the displacement $s_i(t)$ was calculated as a distance between average position of the disclination line at ith point $\langle \mathbf{r}_i \rangle$ and the point where the line perpendicular to average disclination line in ith point crosses the disclination line at time t. The fluctuation modes of the disclination line were obtained by calculating spatial Fourier components of the displacement $s(q_n,t)$. Time autocorrelation function of the Fourier component of the displacement $G^{(1)}(q_n,\tau)=\langle s^*(q_n,t+\tau)s(q_n,t)\rangle$ gives the temporal behavior of the modes. It is a single exponential decaying function with the relaxation rate given by Eq. (2). The amplitude $G^{(1)}(q_n,0)$ is proportional to the average square of the amplitude of a mode with a given q_n. By equipartition theorem the average elastic energy of a mode equals to $\frac{1}{2}k_BT$, where k_B is Boltzman constant and T is the temperature of the system, therefore, the amplitude of the autocorrelation function of such mode is

$$G^{(1)}(q_n,0)=\frac{k_BT}{2l\sigma q_n^2}. \tag{3}$$

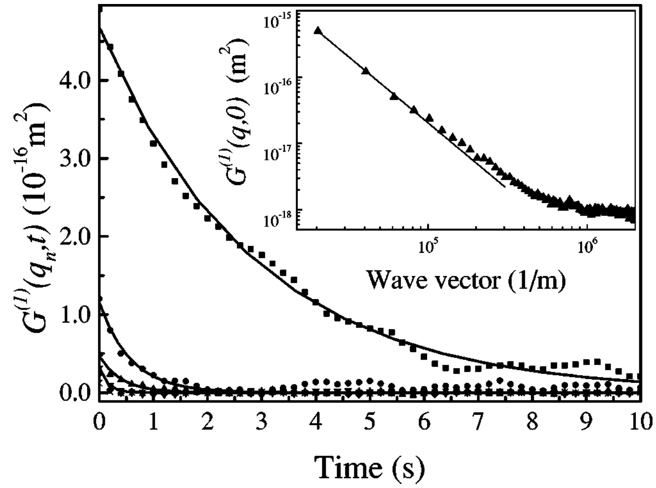

FIG. 2. Time autocorrelation functions as described in the text for six lowest modes. For the first four modes also exponential fits are shown (lines). Inset: Average square amplitude vs wave vector. Line is a fit to inverse quadratic function. The length of disclination is 154 μm and the temperature 303 K.

From the relaxation rates and the value of $G^{(1)}(q_n,0)$ we were able to obtain both the line tension and the viscous drag of disclination lines.

Time autocorrelation functions are shown in Fig. 2. We fitted them with a single exponential function and the ratio σ/R was obtained from the relaxation rate. The decay time can be reliably fit up to about fourth mode and it obeys Eq. (2) very well. In fact, the q^2 dependence of the relaxation rate can be followed to the seventh mode. Higher modes were not accessible due to the limited time and spatial resolution of the microscope images. The line tension was obtained by fitting the q dependence of $G^{(1)}(q_n,0)$ with A/q^2 (Inset of Fig. 2).

The decay of the first mode was in some cases nonexponential, which can be understood if we take into account the effect of the surface. Our model that a disclination line is a massless string, is valid for a disclination line in the bulk. In our experiments line is pinned to the surface. Due to the surface the configuration of the nematic director in the region close to the surface differs from the one in the bulk and also causes the parts of the line close to the surface to be slightly bent towards the surface. That affects the lowest modes the most. Also the pinning of the disclination line to the glass surface is not always perfect, in some cases we observed that the end of the line diffused around some limited region which means that the boundary condition for Eq. (1) is different. That again affects the modes with the lowest wave vector the most. In our analysis we have taken into account only the lines where the time autocorrelation functions were exponential.

Temperature dependence of the measured line tension is shown in Fig. 3 for lines of different lengths. Measured values of line tension are within the experimental error independent of the length except for the longest, i.e., 195 μm line. This deviation is probably due to the surface effect. The above theory is, namely, best valid for the lines that are perpendicular to the glass plates. Using the microscope, how-

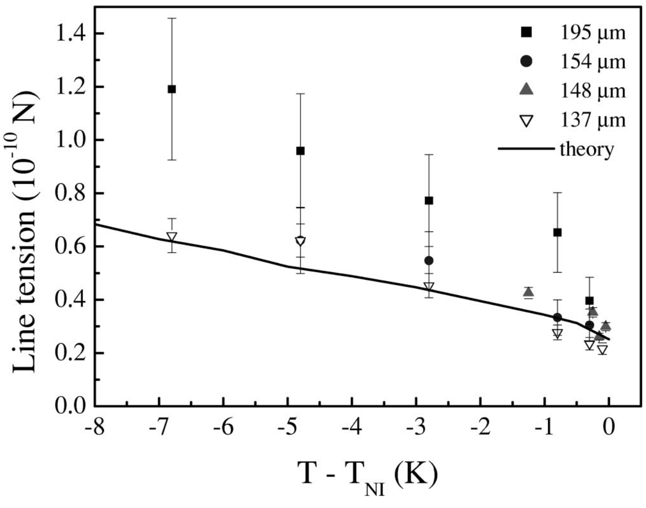

FIG. 3. Temperature dependence of the line tension for disclination lines with different lengths. Line presents the theory [Eq. (4)].

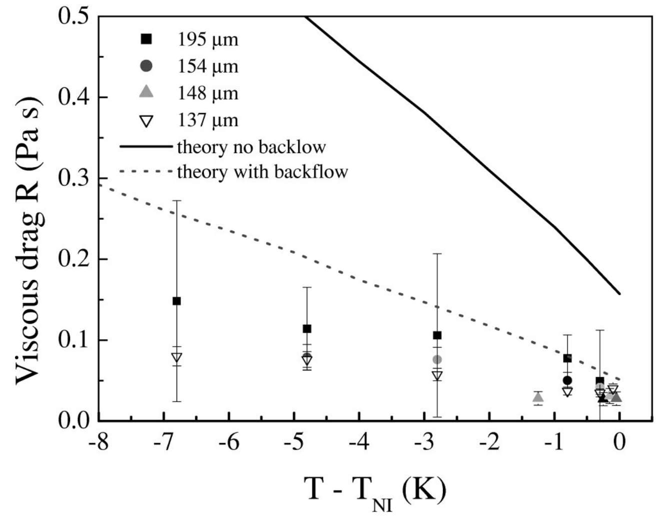

FIG. 4. Temperature dependence of the viscous drag for disclination lines with different lengths. Theoretical prediction [Eq. (5)] with (dotted line) and without (solid line) backflow is also shown.

ever, we cannot study such lines since we only see a point (see for example, Ref. [5]). For longer lines the angle between the line and the glass plate is smaller, i.e., larger part of the line is slightly bent to the surface and therefore the equilibrium configuration of the director and also the effective viscosity are more affected by it. The obtained value of line tension for shorter lines agrees with the theory for the disclination line in bulk [1]

$$\sigma = \pi K s^2 \ln\frac{L}{a} + \sigma_{core}, \tag{4}$$

where K is effective elastic constant, s is strength of the disclination, L is the size of the system, and a is the core size of the disclination line ($L=120$ μm, $a=5$ nm, temperature dependence of K was taken from Ref. [13]). The first term in the expression (4) is obtained by integrating the elastic energy density around the disclination line. The core energy is obtained by assuming that the core is in the isotropic state and its energy density is the difference in free energy of the nematic and isotropic phase. The free energy of the disclination line written as a sum of a core and elastic part is then minimized with respect to the core size a [3]. In expression (4) the contribution of the core of the line is estimated to be about 5–10 % of the total.

Temperature dependence of the measured viscous drag is shown in Fig. 4. Again the measured values are the same within experimental error for all lengths except for the 195 μm line. Similarly as for line tension the proposed theoretical expressions is [8]

$$R = \pi \gamma_1 s^2 \ln\frac{L}{a}, \tag{5}$$

where γ_1 is the rotational viscosity. In expression (5) the coupling between flow and nematic orientation (backflow) is not considered. The measured values are about 15–20 % of the values obtained from the expression (5) with the backflow neglected. It has been shown, however, that the backflow is important for the motion of defects in liquid crystals [6,4,14]. If, as in Ref. [15], we consider γ_1 in Eq. (5) to be an effective viscosity, in which the backflow is approximately taken into account [$\gamma_{eff}=(\gamma_{bend}+\gamma_{twist})/2$, where $\gamma_{twist}=\gamma_1$ and $\gamma_{bend}=\gamma_1-\alpha_2^2/(\eta_b-\gamma_2)$ [15]], our measured values are still about 50 % too low. The measurements of Cladis *et al.* [15] were performed in a different situation, i.e., the disclinations were induced by the geometry of the sample and their motion was induced by an external field, but there also the measurements showed that the viscous force was by about a factor of two smaller than expected. They attributed it to the effect of the external electric field, but possibly the reason in that experiment and in ours is an underestimation of the backflow.

More complicated expressions for the line tension [7] and the viscous drag [9] were proposed, in which ln(L/a) in expressions (4) and (5) is replaced by ln($1.12/\mathcal{E}$) and ln($1.8/\mathcal{E}$), respectively, where $\mathcal{E}$ is the Ericksen number $av\gamma_1/2K$ and v is the velocity of the line. The Ericksen number is the ratio between the characteristic diffusion velocity and the velocity of the disclination line, which tells us that how far from the disclination core the configuration of the order parameter ''feels'' a given displacement of the core of the disclination line in a given time. Such treatment becomes important when the velocity of the line is large compared to $2K/\gamma_1 L$. In our experiments the average maximal velocity of the nth mode of the disclination is $\sim n2\times10^{-8}$ m/s, while $2K/\gamma_1 L \sim 2\times10^{-6}$ s. So the typical velocities in our experiments are much smaller than the critical velocity where the regime changes. In both Refs. [7] and [9] backflow was not considered. Our measurements show that for the description of the line tension simple ''static'' expression [Eq. (4)] is sufficient, while for the viscous drag one has to take into account the backflow, which affects the values of the viscous drag considerably more than expected on the basis of the estimation in Ref. [15].

In conclusion, temperature dependence of the line tension and the viscous drag of a disclination line in nematic liquid crystal were measured using polarization microscopy. Measured values for the line tension agree with the theory already proposed by de Gennes. Measured values of the viscous drag are considerably smaller then predicted by the theory in which backflow was not considered.

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